THERMODYNAMIC EFFICIENCIES REVISITED

Upper and Lower-Bounds to the Efficiency and Coefficient of Performance Based on The 2nd Law of Thermodynamics

By

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Please also see the following related publications:

Mansoori GA, Patel V (1979) Thermodynamic Basis for the Choice of Working Fluids for Solar Absorption Cooling Cycles. Solar Energy Journal J.22: 483-491.

Gomez AL, Mansoori GA (1983) Thermodynamic Equation of State Approach for the Choice of Working Fluids of Absorption Cooling Cycles. Solar Energy J. 31: 557-566

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Thermodynamic Efficiencies Revisited (Upper and Lower-Bounds to the Efficiency and Coefficient of Performance Based on the 2nd Law of Thermodynamics)

ABSTRACT

It is the general practice in engineering to use the Carnot efficiency and Carnot coefficient of performance (COP) as the upper-bounds to efficiency and COP of cycles, respectively. In the present paper through the application of the 2nd law of thermodynamics for irreversible process it is shown that: i) The Carnot efficiency and COP are not necessarily close upper-bounds to the real efficiency and COP ii) The 2nd law of thermodynamics generally gives us efficiency and COP upper-bound relations which are closer to the real efficiency and COP than the Carnot relations. iii) The new efficiency and COP upper-bound relations are in rather simple algebraic forms which can be utilized for energy efficiency analysis of processes. It is also shown that we are now able to produce lower-bounds to efficiency and COP of processes through the use of the 2nd law of thermodynamics. The resulting upper-and lower-bounds to efficiency and COP allow us to analyze a process more precisely than the mere use of the Carnot efficiency and COP relations.

INTRODUCTION

In the analysis of thermodynamic power systems and cooling cycles it has been the general practice to use the ideal Carnot efficiency and Carnot coefficient of performance (COP), respectively. However, due to the ideal nature of Carnot Cycle the resulting efficiency and COP relations provide us only with the upper bounds to the real efficiency and COP which, in most cases, are the highest upper bounds. However, through the application of the second law of thermodynamics for irreversible processes it is possible to derive both upper and lower-bounds to efficiency and COP of thermodynamic systems. The upper-bounds derived in this case will be lower upper-bounds than the Carnot Cycle values. Derivation of both upper and lower bounds to efficiency and COP will allow us to acquire a better understanding about the real performance of a thermodynamic system.

According to thermodynamics of flow processes for an open system with incoming and outgoing flows the first law of thermodynamics can be written in the following form

$$\frac{dE}{dt} - \dot{W}_{in} - \dot{Q}_{in} + \sum_{out} (e + Pv)\dot{M} - \sum_{in} (e + pv)\dot{M} = 0 \qquad (1)$$

In this equation $\frac{dE}{dt}$ is the rate of energy accumulation in the system, $\dot{W}in$ is the rate of work added to the system, $\dot{Q}in$ is the rate of heat added to the system, P is the pressure, v is the specific volume and e is the energy per unit mass of the incoming or outgoing flows. The second law of thermodynamics of a flow process takes the following form for an open system

$$\dot{\rho}_s = \frac{dS}{dt} - \sum_{in} \frac{\dot{Q}}{Te} + \sum_{out} s\dot{M} - \sum_{in} s\dot{M} \geq 0 \qquad (2)$$

In this relation $\dot{\rho}_s$ is the rate of production of entropy, $\frac{dS}{dt}$ is the entropy accumulation rate in the system, s is the entropy per unit mass of the incoming or outgoing flows, and T_e is the temperature of the external heat source or sink.

THERMODYNAMIC POWER CYCLES :

For a thermodynamic power cycle, Fig. 1, considering to be in the steady state and steady flow conditions, application of the second law for the boiler produces the following inequality for $\dot{Q}_B$

$$\dot{Q}_B \leqslant T_H \dot{M} (s_1 - s_2) \tag{3}$$

Application of the second law for the condenser gives us

$$\dot{Q}_C \geqslant T_C \dot{M} (s_4 - s_3) \tag{4}$$

As a result of the above two inequalities, we get

$$\frac{\dot{Q}_C}{\dot{Q}_B} \geqslant \frac{T_C}{T_H} \cdot \frac{(s_4 - s_3)}{(s_1 - s_2)} \tag{5}$$

Considering that the efficiency of the cycle is

$$\eta = 1 - \frac{\dot{Q}_C}{\dot{Q}_B} \tag{6}$$

we conclude from inequality (5) the following upper limit for efficiency:

$$\eta \leqslant 1 - \frac{T_C}{T_H} \cdot \frac{(s_4 - s_3)}{(s_1 - s_2)} \tag{7}$$

The upper limit of efficiency as it is shown by Eq. (7) is lower than the Carnot efficiency, i.e.

$$1 - \frac{T_C}{T_H}\frac{(s_4 - s_3)}{(s_1 - s_2)} \leqslant 1 - \frac{T_C}{T_H} \qquad (8)$$

This is because $(s_4 - s_3) \geqslant (s_1 - s_2)$ and the fact that the Carnot efficiency depends only on the temperatures of the heat source and heat sink and it is independent of the working fluid characteristics. While the upper limit shown by Eq. (7) is dependent on the properties of the working fluid in addition to the temperatures of the heat source and heat sink.

In order to derive a lower limit to the efficiency, we use the first law for the turbine which gives us

$$\dot{W}_T = \dot{M}(h_1 - h_4) \qquad (9)$$

From relations (3) and (9) we conclude

$$\frac{(h_1 - h_4)}{T_H(s_1 - s_2)} \leqslant \eta \qquad (10)$$

The left-hand side of (10) provides us with the lower limit to the efficiency of the cycle. However the theoretical efficiency of the cycle can be calculated from the first law as

$$\eta = \frac{h_1 - h_4}{h_1 - h_2} \qquad (11)$$

Knowing that the actual efficiency of the cycle is even less than Eq. (11) we will have the following upper and lower limits to the actual efficiency of the cycle.

$$\frac{h_1 - h_4}{T_H(s_1 - s_2)} \leqslant \eta_{actual} \leqslant \frac{h_1 - h_4}{h_1 - h_2} \leqslant 1 - \frac{T_C}{T_H}\frac{(s_4 - s_3)}{(s_1 - s_2)} \leqslant 1 - \frac{T_C}{T_H} \tag{12}$$

The above inequalities can be used to calculate the upper and lower limits to the efficiency of the cycle.

THERMODYNAMIC COOLING CYCLES:

There exist two different cooling cycles in thermodynamics. One is the Rankine cooling cycle which, in principle, is the reverse of the thermodynamic power cycles. The other kind of thermodynamic cooling cycle is the absorption refrigeration cycle.

RANKINE COOLING CYCLES: For a Rankine Cooling Cycle, Fig. 2, considering to be in the steady state and steady flow conditions, application of the second law for the evaporator (refrigerator) and the condenser produces the following inequalities:

$$\dot{Q}_E \leqslant T_C \dot{M} (s_4 - s_3) \tag{13}$$

and

$$\dot{Q}_C \geqslant T_o \dot{M} (s_1 - s_2) \tag{14}$$

Considering that the coefficient of performance (COP) of the cycle is defined as

$$COP = \frac{\dot{Q}_E}{\dot{W}_C} = \frac{\dot{Q}_E}{\dot{Q}_C - \dot{Q}_E} , \tag{15}$$

and knowing that from Eq.'s (13) and (14)

$$\dot{Q}_C - \dot{Q}_E \geqslant \dot{M}\left[T_o\,(s_1 - s_2) - T_c\,(s_4 - s_3)\right] , \qquad (16)$$

we get the following upper limit for COP

$$COP \leqslant \frac{T_c}{To\left(\frac{s_1 - s_2}{s_4 - s_3}\right) - T_c} \qquad (17)$$

The upper limit of COP as it is shown by (17) is lower than the Carnot COP, i.e.

$$\frac{T_c}{T_o\left(\frac{s_1 - s_2}{s_4 - s_3}\right) - T_c} \leqslant \frac{T_c}{T_o - T_c} \qquad (18)$$

This is because $(s_1 - s_2) \geqslant (s_4 - s_3)$ and the fact that the Carnot COP depends only, on the temperatures of the heat source and heat sink and it is independent of the working fluid characteristics.

In order to derive a lower limit to the efficiency, we use the first law for the compressor which gives

$$\dot{W}_C = \dot{M}\,(h_1 - h_4) \qquad (19)$$

we also know that

$$T_o\,\dot{M}\,(s_1 - s_2) \leqslant \dot{Q}_C = \dot{Q}_E + \dot{W}_C \qquad (20)$$

By dividing (20) by (19) we get

$$\frac{T_o\,(s_1 - s_2)}{(h_1 - h_4)} - 1 \leqslant COP \qquad (21)$$

The left-hand side of (21) provides us with the lower limit to the COP of the cycle. The theoretical COP of the cycle can be calculated from the first law as

$$COP = \frac{(h_4 - h_3)}{(h_1 - h_4)} \tag{22}$$

Knowing that the actual COP is even less than Eq. (22) we will have the following upper and lower limits to the actual COP of the cycle.

$$T_o \frac{(s_1 - s_2)}{(h_1 - h_4)} - 1 \leqslant (COP)_{actual} \leqslant \frac{h_4 - h_3}{h_1 - h_4} \leqslant \frac{T_c}{To\left(\frac{s_1 - s_2}{s_4 - s_3}\right) - T_c} \leqslant \frac{T_c}{T_o - T_c} \tag{23}$$

The above inequalities can be used to calculate the upper and lower limits to the COP of a Rankine Cooling Cycle.

ABSORPTION COOLING CYCLES (2): The COP of an absorption cycle, Fig. 3, is defined as the ratio of the cooling effect by the evaporator and the heat input to the generator.

$$(COP)_{cycle} = |Q_E| \,/\, |Q_G| \tag{24}$$

According to the first law of thermodynamics the following balance equation exists for the whole cycle.

$$Q_G + Q_E - Q_C - Q_A + W_P = 0, \tag{25}$$

According to the second law of thermodynamics the following relation can be written for the cycle.

$$\Sigma \frac{Qi}{Ti} \leqslant 0$$

or

$$\frac{Q_G}{T_H} + \frac{Q_E}{T_c} - \frac{Q_C}{T_o} - \frac{Q_A}{T_o} \leqslant 0 \tag{26}$$

By assuming W_p, the work input to the liquid pump, negligible as compared to the other terms in Eq. (25) and joining the resulting relation with (26) we get

$$\frac{Q_G}{T_H} + \frac{Q_E}{T_c} - \frac{1}{T_o} (Q_G + Q_E) \leqslant 0 \tag{27}$$

Now, by consideration of the definition of COP, Eq. (24), the above inequality can be rearranged to the following form

$$COP \leqslant \frac{T_c}{T_H} \left(\frac{T_H - To}{To - Tc} \right) \tag{28}$$

This upper limit to COP is identical with the Carnot Cycle COP. According to the first law of thermodynamics for flow systems the following relations exist between the heat and work transfer rates and the properties of the working fluids in a steady state, steady flow condition:

$$\dot{Q}_G = \dot{M}_t h_1 + (\dot{M}_p - \dot{M}_t) h_7 - \dot{M}p\, h_6 , \tag{29}$$

$$\dot{Q}_C = \dot{M}_t (h_1 - h_2), \tag{30}$$

$$\dot{Q}_E = \dot{M}_t (h_4 - h_3); \tag{31}$$

$$\dot{Q}_A = \dot{M}_t h_4 + (\dot{M}_p - \dot{M}_t) h_8 - \dot{M}p\, h_5 \tag{32}$$

$$\dot{W}_p = \dot{M}_p (h_6 - h_5) \tag{33}$$

In the above equations $\dot{M}_t$ is the mass flow rate of refrigerant passing through the throttling valve (I) and $\dot{M}_p$ is the mass flow rate of the solution passing through the liquid pump.

The following relation exists between $\dot{M}_t$ and $\dot{M}_p$

$$\dot{M}_t/\dot{M}_p = (X_A - X_G)/(1 - X_G) \tag{34}$$

where X_A is mass fraction of the refrigerant in the strong liquid phase coming out of absorber and X_G is for the liquid phase coming out of the generator. In deriving this equation it is assumed the vapor which is coming out of the generator is the pure refrigerant.

The following relations also exist for the isenthalpic expansion valves in the cycle

$$h_2 = h_3 \quad \text{and} \quad h_7 = h_8 \tag{35}$$

by assuming $\dot{W}_p$ the power input to the liquid pump, negligible the following relation will be also derived

$$h_6 = h_5 \quad . \tag{36}$$

Based on the above equations the COP of the cycle can be defined with respect to flow properties by the following relation

$$(COP)_{cycle} = \frac{|\dot{Q}_E|}{|\dot{Q}_G|} = \frac{h_4 - h_2}{(h_1 - h_7) + \left(\frac{1 - X_G}{X_A - X_G}\right)(h_7 - h_5)} \tag{37}$$

According to the second law of thermodynamics for open systems the following relations also exist between the heat transfer rates and the properties of the working fluids in a steady-state absorption cycle:

$$|\dot{Q}_G| \leqslant T_H \left\{\dot{M}_t (s_1 - s_7) + \dot{M}_p (s_7 - s_6)\right\} \tag{38}$$

$$|\dot{Q}_C| \geqslant T_O \; \dot{M}_t (s_1 - s_2) \tag{39}$$

$$|\dot{Q}_E| \leqslant T_c \; \dot{M}_t (s_4 - s_3) \tag{40}$$

$$|\dot{Q}_A| \geqslant T_O \left\{\dot{M}_t (s_4 - s_8) + \dot{M}_p (s_8 - s_5)\right\} \tag{41}$$

In the above relations the equality sign is for the reversible case and the inequality sign is for the irreversible case. By joining eqns. (38) and (40), we get

$$|\dot{Q}_G| + |\dot{Q}_E| \leqslant \dot{M}_t \left\{ T_H (s_1 - s_7) + Tc (s_4 - s_3) \right\} + \dot{M}_p T_H (s_7 - s_6) \quad (42)$$

Also, by joining eqns. (39) and (41) we get

$$|\dot{Q}_c| + |\dot{Q}_A| \geqslant \dot{M}_t \left\{ To (s_1 - s_2) + To (s_4 - s_8) \right\} + \dot{M}_p To (s_8 - s_5) \quad (43)$$

Again, by assuming that $\dot{W}_p$ is negligible as compared to the other terms in eqn. (25) we can write

$$|\dot{Q}_G| + |\dot{Q}_E| = |\dot{Q}_C| + |\dot{Q}_A| \quad . \quad (44)$$

From eqns. (43) - (44) we conclude that

$$\dot{M}_t \left\{ To (s_1 - s_2) + To (s_4 - s_8) \right\} + \dot{M}_p To (s_8 - s_5) \leqslant |\dot{Q}_G| + |\dot{Q}_E| \leqslant \dot{M}_t \left\{ T_H (s_1 - s_7) + Tc (s_4 - s_3) \right\} + \dot{M}_p T_H (s_7 - s_6). \quad (45)$$

By dividing relation (45) by $|Q_G|$ and consideration of the definition of (COP) cycle as given by eqn. (24) we get the following relation

$$LL \leqslant (COP)_{cycle} \leqslant UL \quad (46)$$

where the lower limit, LL, is in the following form

$$LL = \frac{(\dot{M}_t/\dot{M}_p) \left\{ To (s_1 - s_2) + To (s_4 - s_8) \right\} + To (s_8 - s_5)}{(\dot{M}_t/\dot{M}_p) (h_1 - h_7) + (h_7 - h_5)} - 1 , \quad (47)$$

and the upper limit, UL, is in the following form

$$UL = \frac{(\dot{M}_t/\dot{M}_p) \left\{ T_H (s_1 - s_7) + Tc (s_4 - s_3) \right\} + T_H (s_7 - s_6)}{(\dot{M}_t/\dot{M}_p) (h_1 - h_7) + (h_7 - h_5)} - 1 . \quad (48)$$

Relations (46)-(48) can be used to calculate the upper and lower limits of the (COP) cycle at different working conditions. This means that the (COP) cycle will not go out of the bounds specified by eqn. (46). However it should be pointed out that the actual COP of the cycle is even less than the COP as calculated by eqn. (37). Also with the understanding that inequality (28) gives the upper limit of COP regardless of the working fluids under consideration and since it is for a Carnot cycle, it is always higher than UL as given by (48). As a result, we can write

$$LL \leqslant (COP)\ \text{actual} \leqslant (COP)\ \text{eqn. (37)} \leqslant UL \leqslant \frac{T_C}{T_H}\left(\frac{T_H - T_o}{T_o - T_C}\right) \quad (49)$$

The above inequalities can be used to calculate the upper and lower limits to the COP of an absorption refrigeration (cooling) cycle.

<u>CONCLUSION:</u> The inequalities produced in this report provide us with upper and lower limits to efficiencies and COP's of power generating and cooling cycles. There are several advantages associated with these inequalities. One advantage is that we are able to produce, both, upper and lower limits to the actual efficiency and COP of a cycle which is quite useful for a prior design analysis of thermodynamic cycles. The other advantage is with regard to studies on specification of working fluids for cycles which would result in the optimum performance of cycles. For example, in studies of the choice of working fluids for solar-assisted absorption cycles it is shown that [2] inequality (49) is quite instrumental in comparation analysis of different absorbent-refrigerant combinations. Over all, the inequalities presented in this report could be quite useful in design of thermodynamic power cycles from the point of view of energy conservation based on the first and second laws of thermodynamics.

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1. Callen, H.B., "Thermodynamics", J. Wiley & Sons, Inc., New York, N.Y. (1960).

2. Mansoori, G.A. and Patel, V., Solar Energy, Vol. 22, pp. 483-491 (1979).

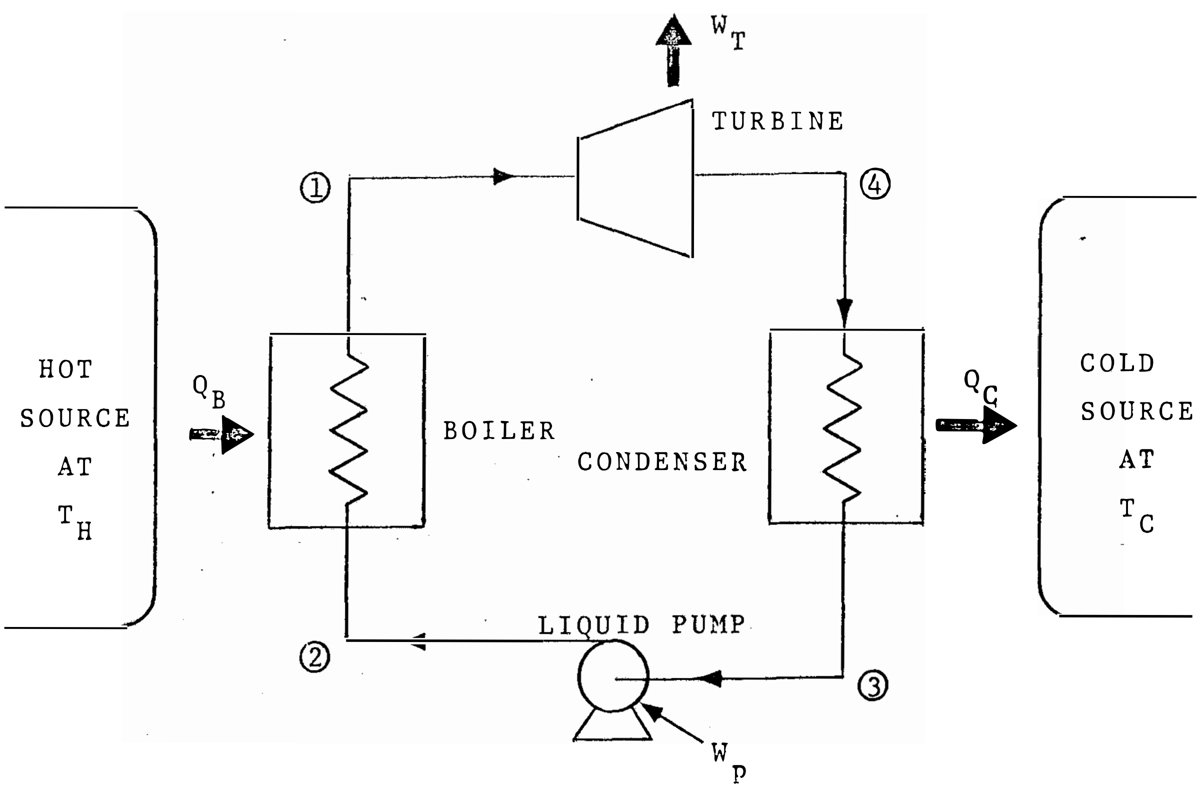

Fig. 1. Thermodynamic Power Cycle

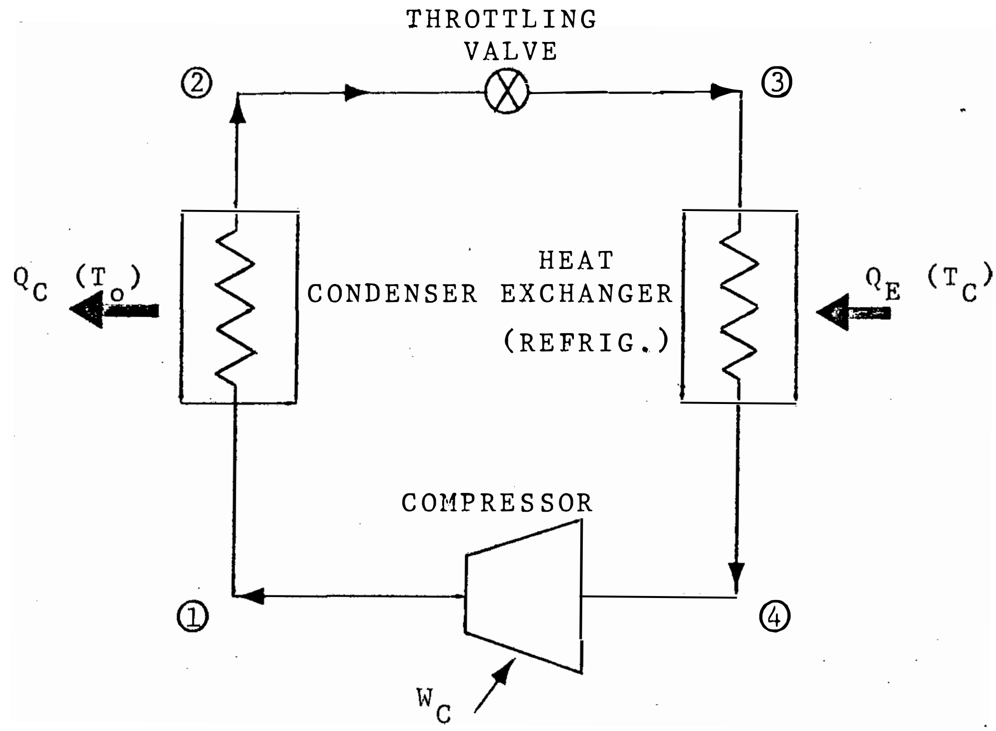

Fig. 2. Rankine Cooling Cycle

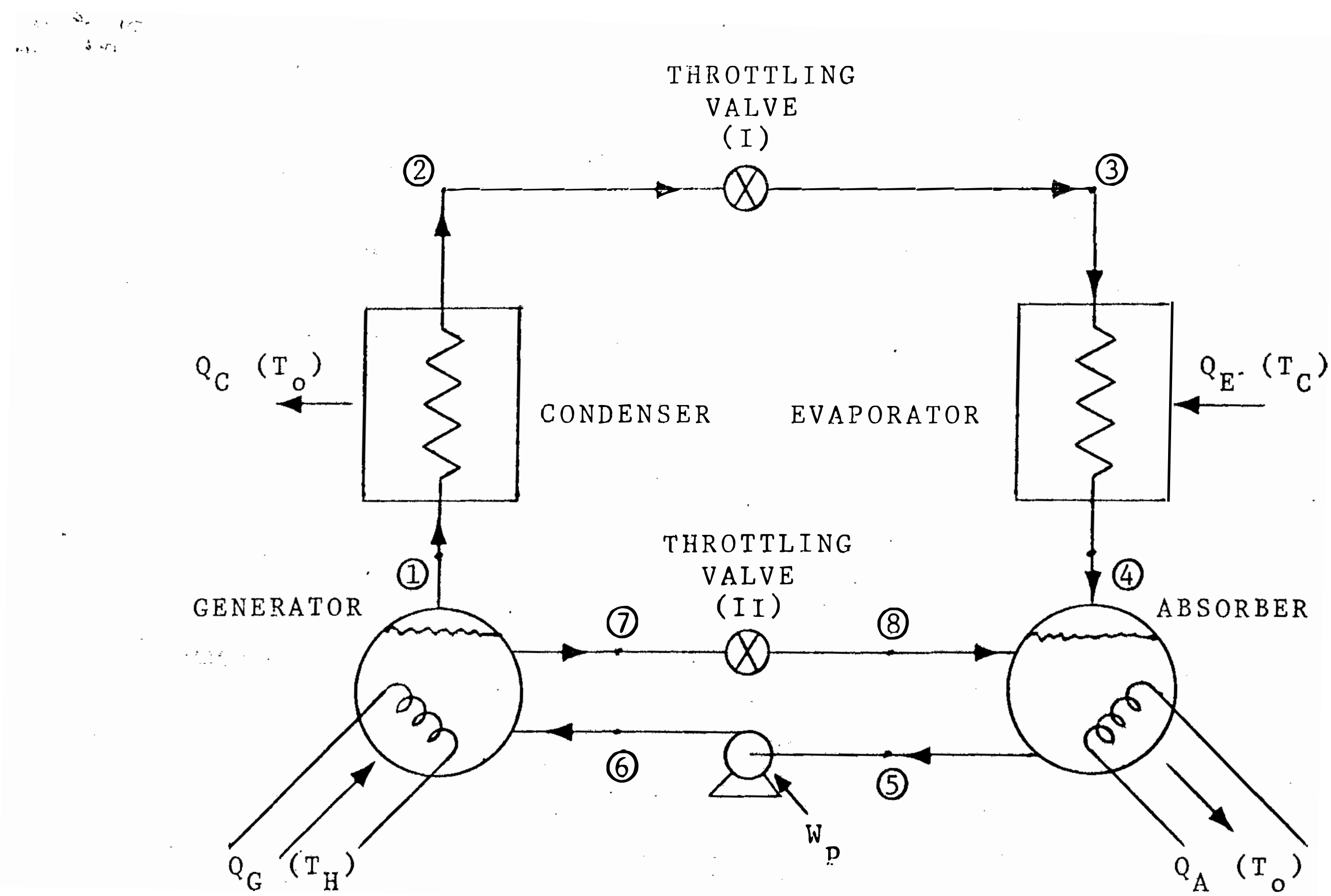

Fig. 3. Absorption Cooling Cycle.